conditional sentences and causal reasoning

seminar 3: An asymmetric notion of inference

Katrin Schulz
ILLC
University of Amsterdam
Conditional Sentences and Causal Reasoning

Course Plan

Tuesday

Lecture 1: The logic of conditionals: The standard view

Tutorial 1: Challenges for the similarity approach

Lecture 2: Bayes Nets and Causal Bayes Nets

Tutorial 2: Counterfactuals as Interventions

Wednesday

Thursday

Friday

Seminar 1 (Practicum, part 1): 3 challenges for the framework (I will first introduce the challenges individually, then you can choose and work in groups on one of them for 40 minutes)

Seminar 2: The relation between the similarity approach and the causal approach

Seminar 3: Using Logic Programming to model causal inferences

Seminar 4 (Practicum, part 2): presentations and discussion (each group will shortly present their ideas, we will discuss them and I will comment on the state of the arts on each of the challenges)
Plan today

• Introduce an asymmetric notion of inference using Logic Programming
• Look at an application: Causal Premise Semantics
• Reflections: Conditionals and Causality
Introduction

• We saw that for recursive models Pearl’s approach to counterfactuals is a specific case of a similarity approach (or premise semantics)
Introduction

• We saw that for recursive models Pearl’s approach to counterfactuals is a specific case of a similarity approach (or premise semantics)

• according to this order every fact about the world is relevant, but to a different degree
Introduction

- Compare the order to Lewis’ order: similarities and differences?
- What is responsible for the asymmetry in reasoning?
- What about Pearl? How does he get his asymmetry? Where is it in the model? How is it used?
similarity & structural equations

but which premises and how are they ordered?

\[ Y_1 = X_1 \land X_2 \]
\[ Z = \neg Y_1 \land Y_2 \]
What about Pearl? How does he get his asymmetry? Where is it in the model? How is it used?
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Introduction

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Introduction

• What about Pearl? How does he get his asymmetry? Where is it in the model? How is it used?
similarly & structural equations

**Question:**
Can’t we have an asymmetric notion of reasoning

**Result:**
Yes, we can!
Using **Logic Programming**
→ with this we can give a causal version of Veltman’s premise semantics
→ but there is a difference with Pearl and that difference seems to matter …
Logic Programming and Causal Inference
We need a causal notion of consequence: reasoning from cause to effect (i.e. it should be asymmetric).

A CAUSAL NOTION OF CONSEQUENCE

- Primitive facts
- Model of causal dependencies
- Causally entailed facts

✓ Logic Programming
The idea: A fixed point construction

- we define the facts causally entailed by $\Sigma$ as the sentences true in a particular “minimal” model of $\Sigma$ and $D$
- this minimal model is constructed by iteratively applying a monotone operator $T$ to models of $\Sigma$
The idea: A fixed point construction

A CAUSAL NOTION OF CONSEQUENCE
The idea: A fixed point construction

A CAUSAL NOTION OF CONSEQUENCE

The model $\mathbf{D}$

The primitive facts $\Sigma$
The idea: A fixed point construction

The model $D$
- the primitive facts $\Sigma$
- causally entailed facts
A CAUSAL NOTION OF CONSEQUENCE

The idea: A fixed point construction

The model $D$

the primitive facts $\Sigma$

causally entailed facts
The idea: A fixed point construction

A CAUSAL NOTION OF CONSEQUENCE

The model $D$

The primitive facts $\Sigma$

Causally entailed facts
A CAUSAL NOTION OF CONSEQUENCE

The idea: A fixed point construction

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**A CAUSAL NOTION OF CONSEQUENCE**

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$\Sigma^*$
A CAUSAL NOTION OF CONSEQUENCE

Standard definition
The operator $\tau_D$ associated with a structural model $D$ is a function on the set of three-valued models. For every proposition letter $p$ the value $\tau_D(M)(p)$ is defined as follows.

1. $\tau_D(M(p)) = 1$ if there is an equation $p = \Phi$ in $D$ such that $M(\Phi) = 1$.

2. $\tau_D(M)(p) = 0$ if for all equations $p = \Phi$ in $D$ it holds $M(\Phi) = 0$.

3. $\tau_D(M)(p) = u$, otherwise.
A CAUSAL NOTION OF CONSEQUENCE

Stenning & van Lambalgen

The operator $\tau_D$ associated with a structural model $D$ is a function on the set of three-valued models. For every proposition letter $p$ the value $\tau_D(M)(p)$ is defined as follows.

1. $\tau_D(M(p)) = 1$ if there is an equation $p = \Phi$ in $D$ such that $M(\Phi) = 1$.

2. $\tau_D(M)(p) = 0$ if there is an equation $p = \Phi$ in $D$ and for all such clauses $M(\Phi) = 0$.

3. $\tau_D(M)(p) = u$, otherwise.
The new definition
The operator $\tau_D$ associated with a structural model $D$ is a function on the set of three-valued models. For every proposition letter $p$ the value $\tau_D(M)(p)$ is defined as follows.

1. If $M(p) \neq u$, then $\tau_D(M)(p) = M(p)$.

2. If $M(p) = u$, then
   a. $\tau_D(M)(p) = 1$ if there is an equation $p = \phi$ in $D$ such that $M(\phi) = 1$.
   b. $\tau_D(M)(p) = 0$ if there is an equation $p = \phi$ in $D$ and for all such clauses $M(\phi) = 0$. 

A CAUSAL NOTION OF CONSEQUENCE
A CAUSAL NOTION OF CONSEQUENCE

Fact
The operator $T_D$ has a fixed point. The least fixed point $s^*_\Sigma$ of $T_D$ applied to a situation $s_\Sigma$ is reached in finitely many steps.

This least fixed point is the model we use to define the causal consequences of $F$

$$F \mid_{\Sigma L} \psi \iff s^*_\Sigma(\psi) = 1$$
... AND ITS NON-MONOTONICITY

- we use the semantics of logic programming
- but no negation as failure!!!
- still non-monotonic!!!
... AND ITS NON-MONOTONICITY

Illustration:

the model $D$

$X_1 \land X_2 \leftrightarrow Y_1$

$Y_1 \land Y_2 \leftrightarrow Z_1$
... AND ITS NON-MONOTONICITY

Illustration:

- The model $D$
  - $X_1 \land X_2 \leftrightarrow Y_1$
  - $Y_1 \land Y_2 \leftrightarrow Z_1$

- The primitive facts $\Sigma$

- $X$, $X$, $X$, $X$
- $Y$
- $Y$
- $Z$
Illustration:

\[
\begin{align*}
X_1 & \quad X_2 \leftrightarrow Y_1 \\
Y_1 & \quad Y_2 \leftrightarrow Z_1
\end{align*}
\]

... AND ITS NON-MONOTONICITY
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... AND ITS NON-MONOTONICITY
Illustration:

\[
X_1 \land X_2 \leftrightarrow Y_1 \\
Y_1 \land Y_2 \leftrightarrow Z_1
\]

the model \( D \)

\[
\begin{array}{cccccccc}
X & X & X & X & Y & Y & Y & Z \\
\hline
s & 1 & 1 & u & u & 0 & 1 & u \\
T & 1 & 1 & u & u & 0 & 1 & 0 \\
\end{array}
\]

... AND ITS NON-MONOTONICITY
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Illustration:

The model D

\[ X_1 \land X_2 \leftrightarrow Y_1 \]
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two perspectives on defeasible causal laws

laws defeated by direct information that the expected effect does not hold

laws defeated by information about abnormal circumstances, abnormal effect is derived

van Lambalgen et al.
Definition: For a set of literals $\Sigma$ (the facts), $\Sigma \models_P C$ iff $C$ is true on the fixed point $\tau_P^*(\Sigma)$ of $\tau_P$ reached by iterating $\tau_P$ starting from $\Sigma$.

Results: 1. For all $M$ we have $M \leq \tau_P(M)$.
2. The iterative application of $\tau_P$ to a model terminates in a fixed point of $\tau_P$ after finitely many steps.
A CAUSAL NOTION OF CONSEQUENCE

A remark

- The operator $\tau_P$ is made for logic programs, not for structural models — thus, a structural model needs to be translated into a logic program first.

- Not all (recursive) structural models can be translated into logic programs: the right side of the equations needs to be equivalent with a conjunction of literals.

- But we can also express things in logic programs that Pearl cannot express: we can have more than one program clause expressing the relation between a variable and its parents.
Causal Premise Semantics
SIMILARITY APPROACH
ANOTHER APPROACH: PREMISE SEMANTICS

A conditional sentence ‘If A then C’ is true iff:

\[
\begin{array}{c}
A \\
\Rightarrow \\
C \\
\end{array}
\]
ANOTHER APPROACH: PREMISE SEMANTICS

A conditional sentence ‘If A then C’ is true iff:

\[
A + \text{Facts of } w_0 \Rightarrow_L C
\]

maximal consistent subset

premise set \( P(w_0) \)

general laws

special case of the similarity approach:

\[w_1 \leq w_2 \text{ iff } \{p \in P(w_0) \mid w_1 \models p\} \supseteq \{p \in P(w_0) \mid w_2 \models p\}\]
VELTMAN’S PREMISE SEMANTICS

A conditional sentence ‘If A then C’ is true iff:

\[
A \quad + \quad \text{Facts of } w_0 \quad \Rightarrow_L \quad C
\]

maximal consistent subset
premise set \( P(w_0) \)
general laws
A conditional sentence ‘If $A$ then $C$’ is true iff:

\[
\begin{array}{cccc}
A & + & \text{Facts of } w_0 & \Rightarrow_L & C \\
\uparrow & & \uparrow & & \uparrow \\
\text{maximal} & \text{basis} & \text{general} & & \\
\text{consistent} & B(w_0) & \text{laws} & & \\
\text{subset} & & & & \\
\end{array}
\]

\[
\text{Basis}(w_0) = \text{minimal set of primitive facts of } w_0 \text{ from which, given } L, \text{ all other facts of } w_0 \text{ can be derived.}
\]

\[\Rightarrow \text{Particular variant of Premise Semantics}\]
**Definition: basis**

The basis $B_L(w_0)$ of the evaluation world $w_0$ is the minimal set of primitive facts (literals) of $w_0$ from which all other facts of $w_0$ follow.

Veltman '05

causally follow.
There is a court, an officer, a rifleman and a prisoner. If the court orders the execution of the prisoner, the officer will give a signal to the rifleman, the rifleman will shoot and the prisoner will die.
CAUSAL PREMISE SEMANTICS

court (C) orders

C ↔ O

officer (O) signals

O ↔ R

rifleman shoots (R)

R ↔ P

prisoner dies (P)
CAUSAL PREMISE SEMANTICS

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CAUSAL PREMISE SEMANTICS

- Court (C) orders
- Officer (O) signals
- Rifleman shoots (R)
- Prisoner dies (P)

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**CAUSAL PREMISE SEMANTICS**

![Causal Premise Diagram]

- **court (C) orders**
- **officer (O) signals**
- **rifleman shoots (R)**
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<th>P</th>
</tr>
</thead>
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<td>w0</td>
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<td>w1</td>
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<tr>
<td>w3</td>
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<td>1</td>
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</tr>
<tr>
<td>w4</td>
<td>1</td>
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</tbody>
</table>
A conditional sentence ‘If A then C’ is true iff:

\[ W_D(A) \cup_C B_D(w_0) \models_D C \]
CAUSAL PREMISE SEMANTICS

Definition: witness sets
A witness set $s(A)$ of sentence $A$ is a minimal set of primitive facts that forces $A$ no matter of the causal circumstances, i.e. all consistent extension of $s(A)$ causally entail $A$. 
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**Examples:**

A is atomic $\Rightarrow$ unique witness set: \{A\}
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A witness set $s(A)$ of sentence $A$ is a minimal set of primitive facts that forces $A$ no matter of the causal circumstances, i.e. all consistent extension of $s(A)$ causally entail $A$.

**Examples:**

$A$ is $p \land q$  $\Rightarrow$ unique witness set: $\{p, q\}$
Definition: **witness sets**

A witness set $s(A)$ of sentence $A$ is a minimal set of primitive facts that forces $A$ no matter of the causal circumstances, i.e. all consistent extension of $s(A)$ causally entail $A$.

**Examples:**

$A$ is $p \lor q \Rightarrow$ two witness sets: $\{p\}$, $\{q\}$
CAUSAL PREMISE SEMANTICS

A conditional sentence ‘If A then C’ is true iff:

Antecedent + Facts of $w_0$ $\Rightarrow_L$ Consequent

- Causal Premise Semantics

  - No matter how you force A in $w_0$ (by intervention), C will causally follow.

\[
W_D(A) \cup C_B_D(w_0) \models_D C
\]
CAUSAL PREMISE SEMANTICS

**Question:** Is this version of premise semantics still a special case of minimal world reasoning?
Can we recast this approach in terms of the Lewis/Stalnaker similarity approach?

**Answer:** YES!
conditionals and causality
“counterfactuals are generated and evaluated by symbolic operations on a model that represents an agent’s beliefs about functional relationships in the world” [Pearl 2013, p. 977]
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King Ludwig of Bavaria likes to spend his weekends in Leoni Castle. Whenever the king is in the castle, the lights will be on and the royal flag will be up. A traveler watches the castle from a distance and sees that the lights are on. The flag, however, is not up. He says …

(2) If the flag had been up, the king would have been in the castle.
"counterfactuals are generated and evaluated by symbolic operations on a model that represents an agent’s beliefs about functional relationships in the world" [Pearl 2013, p. 977]

It is a simple fact of basic math that if you add two natural numbers that are both even or uneven, the sum will be even. If one of the numbers is even and the other uneven, their sum is uneven. Suppose you are explaining this fact to someone and you wrote down as an example $3 + 4 = 7$. You say …

(3) If the first number had been even, the result would have been even.
IS THIS REALLY ABOUT CAUSALITY?

• conditionals exploit certain invariant relationships, certain asymmetric dependencies

• what unifies these relationships is that the expressed dependency is one of manipulation and control:
  ‣ *A stands in this relation to B if manipulating A will change B in a systematic way;*
  ‣ *by manipulating A one can control B*

• an invariant relationship with these properties I call a *causal relation*
References

primary texts:

- van Lambalgen & Hamm (2005), The proper treatment of events. John Wiley and Sons.